ECS 455 Chapter 1

Introduction & Review

1.3 Wireless Channel (Part 1)

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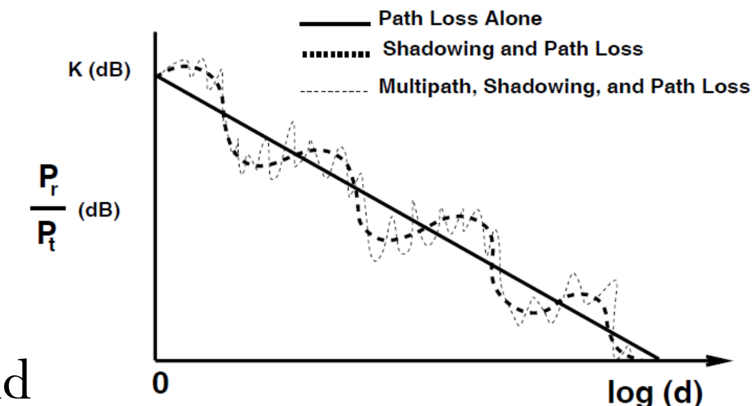
Wireless Channel

- **Large-scale** propagation effects
  1. Path loss
  2. Shadowing
  - Typically frequency independent



[Goldsmith, 2005, Fig 2.1]

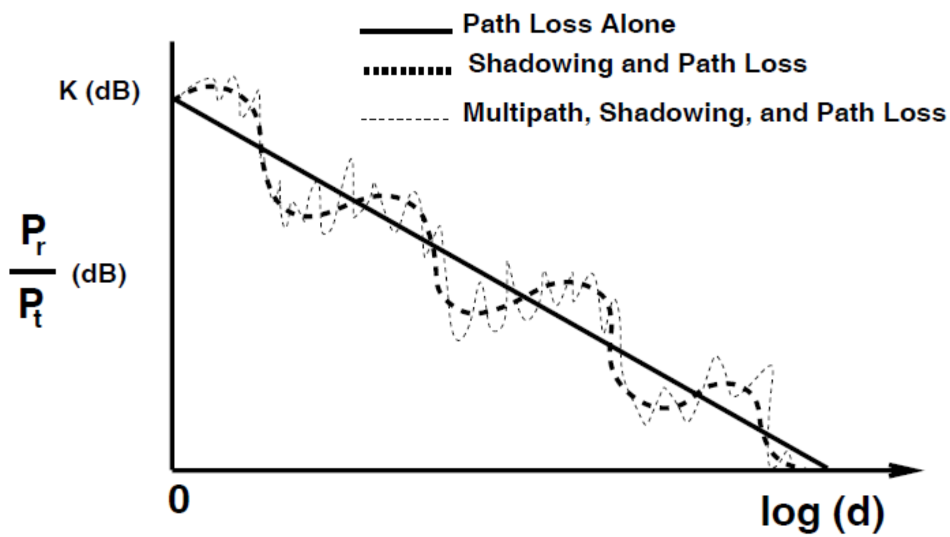
- **Small-scale** propagation effects
  - Variation due to the constructive and destructive addition of **multipath** signal components.
  - Occur over very short distances, on the order of the signal wavelength.

$$\boxed{\lambda = \frac{c}{f}} \quad c \approx 3\times 10^8 \text{ [m/s]}$$

$f = 3$ GHz → $\lambda = 0.1$ m

Path loss

- Caused by
  - dissipation of the power radiated by the transmitter
  - effects of the propagation channel
- Models generally assume that it is the same at a given transmit-receive distance.
- Variation occurs over **large distances** (100-1000 m)



[Goldsmith, 2005, Fig 2.1]

Path Loss (PL)

$$P_L = \frac{\text{Transmitted power}}{\text{Average received power}} = \frac{P_t}{P_r}$$

Averaged over any random variations

- **Free-Space Path Loss**:

$$\frac{P_r}{P_t} \propto \frac{1}{d^2}$$

  - P_r falls off inversely proportional to the square of the distance d between the Tx and Rx antennas.

- **Simplified** Path Loss Model:

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}$$

To be discussed

(Path loss of the free-space model)

Friis Equation (Free-Space PL)

- One of the most fundamental equations in antenna theory

1 for non-directional antennas

$$\frac{P_r}{P_t} = \left(\frac{\sqrt{G_{Tx} G_{Rx}}\,\lambda}{4\pi d} \right)^2 = \left(\frac{\sqrt{G_{Tx} G_{Rx}}\,c}{4\pi d f} \right)^2$$

- More power is lost at higher frequencies.

$$0.7\text{ GHz} \longrightarrow 2.4\text{ GHz} \longrightarrow 5\text{ GHz} \longrightarrow 60\text{ GHz}$$

10.7 dB loss	6.4 dB loss	21.6 dB loss
$20\log_{10}\frac{2.4}{0.7}$	$20\log_{10}\frac{5}{2.4}$	$20\log_{10}\frac{60}{5}$

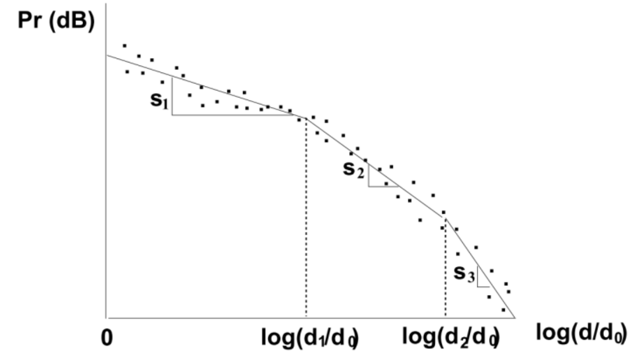
- Some of these losses can be offset by reducing the maximum operating range.
  - The remaining loss must be compensated for by increasing the antenna gain.

Path Loss Models

- Analytical models
  - Maxwell's equations
  - Ray tracing

Prohibitive (complex, impractical) Need to know/specify "almost everything" about the environment.

- Empirical models: Developed to predict path loss in typical environment.
  - Okumura
  - Hata
  - COST 231
    - by EURO-COST (EUROpean Cooperative for Scientific and Technical research)
  - Piecewise Linear (Multi-Slope) Model
- Tradeoff: Simplified Path Loss Model



Indoor Attenuation Factors

- Building penetration loss: 8-20 dB (better if behind windows)
- Attenuation between floors
  - @ 900 MHz
    - 10-20 dB when the Tx and Rx are separated by a single floor
    - 6-10 dB per floor for the next three subsequent floors
    - A few dB per floor for more than four floors
  - Typically worse at higher frequency.
- Attenuation across floors

Partition Type	Partition Loss in dB
Cloth Partition	1.4
Double Plasterboard Wall	3.4
Foil Insulation	3.9
Concrete wall	13
Aluminum Siding	20.4
All Metal	26

[Goldsmith, 2005, Sec. 2.5.5]

Simplified Path Loss Model

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}$$

$$\underset{[\text{dB}]}{10\log_{10}\frac{P_r}{P_t}} = \left(10\log_{10} K d_0^{\gamma}\right) - 10\gamma\log_{10} d$$

Captures the essence of signal propagation without resorting to complicated path loss models, which are only approximations to the real channel anyway!

- K is a unitless constant which depends on the antenna characteristics and the average channel attenuation
  - $\left(\frac{\lambda}{4\pi d_0}\right)^2$ for free-space path gain at distance d_0 assuming omnidirectional antennas
- d_0 is a reference distance for the antenna far-field
  - Typically 1-10 m indoors and 10-100 m outdoors.

(Near-field has scattering phenomena.)

- γ is the **path loss exponent**.

[Goldsmith, 2005, Table 2.2]

Path Loss Exponent γ

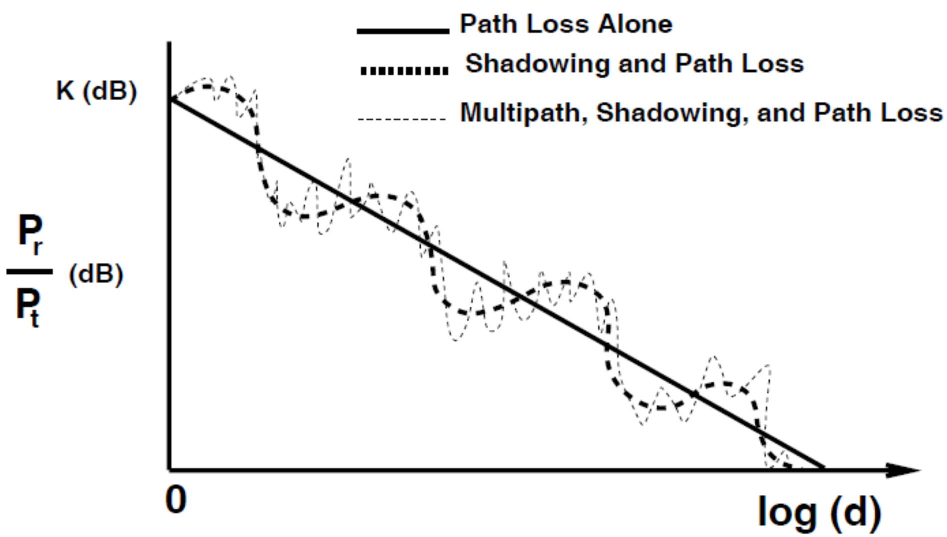
- 2 in free-space model
- 4 in two-ray model
  [Goldsmith, 2005, eq. 2.17]
- Cellular: 3.5 – 4.5
  [Myung and Goodman, 2008 , p 17]
- Larger @ higher freq.
- Lower @ higher antenna heights

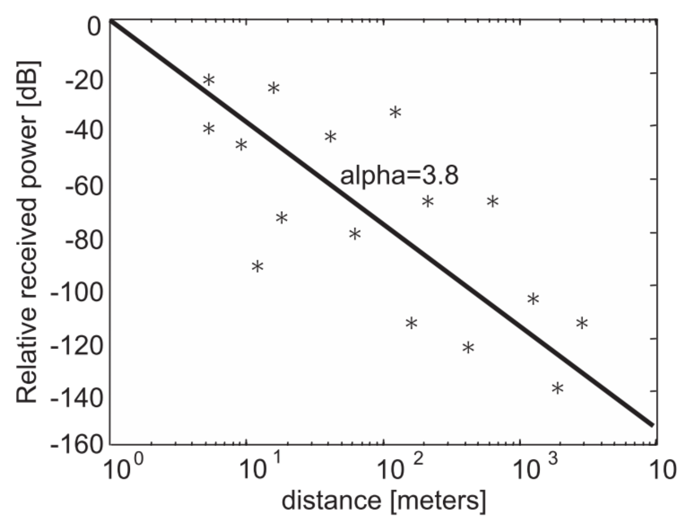
Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

Shadowing (or Shadow Fading)

- Additional attenuation caused by **obstacles** (**large objects** such as buildings and hills) between the transmitter and receiver.
  - Think: cloud blocking sunlight
- Attenuate signal power through absorption, reflection, scattering, and diffraction.
- Variation occurs over distances proportional to the length of the obstructing object (**10-100 m** in outdoor environments and less in indoor environments).

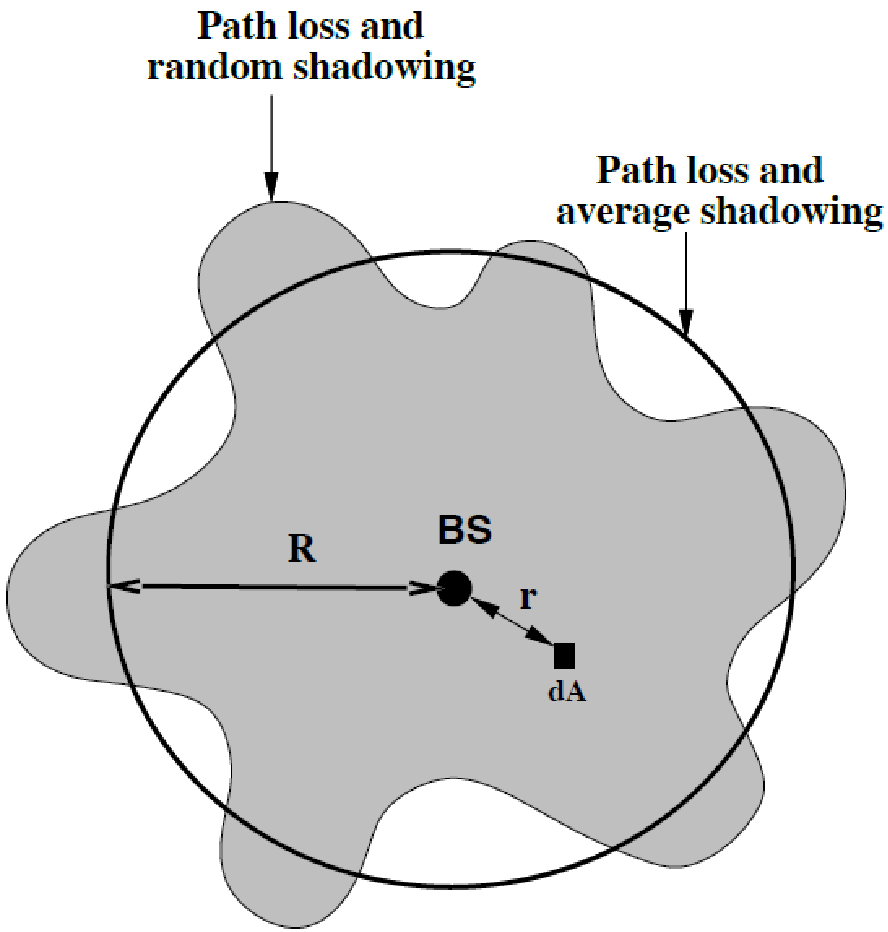
[Goldsmith, 2005, Fig 2.1]





[Myung and Goodman, 2008, Fig 2.1]

Contours of Constant Received Power



[Goldsmith, 2005, Fig 2.10]

Log-normal shadowing

- Random variation due to blockage from objects in the signal path and changes in reflecting surfaces and scattering objects → random variations of the received power at a given distance

4 – 13 dB with higher values in urban areas and lower ones in flat rural environments.

$$10\log_{10}\frac{P_t}{P_r} \sim \mathcal{N}\left(\mu,\sigma^2\right)$$

in dB

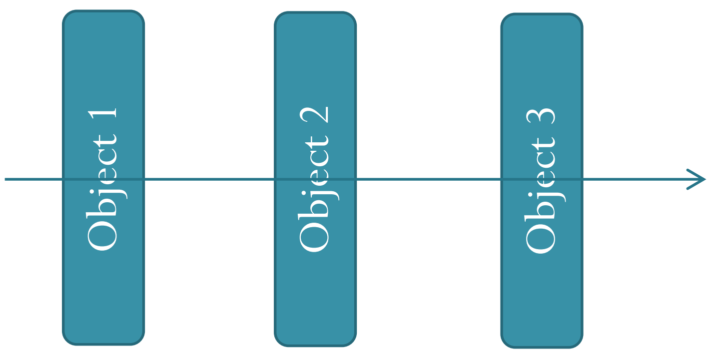
- This model has been confirmed empirically to accurately model the variation in received power in both outdoor and indoor radio propagation environments.

[Erceg et al, 1999] and [Ghassemzadeh et al, 2003]

Log-normal shadowing (motivation)

- Location, size, dielectric properties of the blocking objects as well as the changes in reflecting surfaces and scattering objects that cause the random attenuation are generally unknown

  $\Rightarrow$ statistical models must be used to characterize this attenuation.
- Assume a large number of shadowing objects between the transmitter and receiver



Without the objects, the attenuation factor is $K\left(\frac{d_0}{d}\right)^{\gamma}$.

Each object introduce extra power loss factor of α_i.

So,

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma} \prod_i \alpha_i$$

$$10\log_{10}\frac{P_r}{P_t} = 10\log_{10} K\left(\frac{d_0}{d}\right)^{\gamma} + \underbrace{\sum_i 10\log_{10}\alpha_i}_{\text{By CLT, this is approximately Gaussian}}$$

PDF of Lognormal RV

- Consider a random variable

$$R = \frac{P_t}{P_r}$$

- Suppose

$$10 \log_{10} R \sim \mathcal{N}\left(\mu, \sigma^2\right)$$

- Then,

$$f_R\left(r\right) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} \frac{10}{\ln 10} \frac{1}{r} e^{-\frac{1}{2}\left(\frac{\left(10 \log r\right) - \mu}{\sigma}\right)^2}, & r > 0 \\ 0, & \text{otherwise.} \end{cases}$$

PDF of Lognormal RV (Proof)

Suppose $c\log_b Y \sim \mathcal{N}\left(\mu,\sigma^2\right)$.

Let $X = c\log_b Y$. Note that $X = c\log_b Y = \dfrac{c}{\ln b}\ln\left(Y\right) = k\ln\left(Y\right)$.

Then, $Y = e^{\frac{X}{k}}$ where $k = \dfrac{c}{\ln b}$.

Recall, from ECS315 that to find the pdf of $Y = g(X)$ from the pdf of X, we first find the cdf of Y and then differentiate to get its pdf:

$$F_Y\left(y\right) = P\left[Y \le y\right] = P\left[e^{\frac{X}{k}} \le y\right] = P\left[X \le k\ln\left(y\right)\right] = F_X\left(k\ln\left(y\right)\right).$$

$$f_Y\left(y\right) = \frac{d}{dy}F_X\left(k\ln\left(y\right)\right) = \frac{k}{y}f_X\left(k\ln\left(y\right)\right) = \frac{1}{\sqrt{2\pi}\sigma}\frac{k}{y}e^{-\frac{1}{2}\left(\frac{k\ln\left(y\right)-\mu}{\sigma}\right)^2}.$$

PDF of Lognormal RV (Proof)

Suppose $c\log_b Y \sim \mathcal{N}\left(\mu,\sigma^2\right)$.

Let $X = c\log_b Y$. Note that $X = c\log_b Y = \dfrac{c}{\ln b}\ln\left(Y\right) = k\ln\left(Y\right)$.

Then, $Y = e^{\frac{X}{k}}$ where $k = \dfrac{c}{\ln b}$.

Alternatively, to find the pdf of $Y = g(X)$ from the pdf of X, when g is monotone, we may use the formula:

$$f_X\left(x\right)dx = f_Y\left(y\right)dy \quad \Longrightarrow \quad \boxed{f_Y\left(y\right) = \frac{dx}{dy} f_X\left(x\right)}$$

This gives $f_Y\left(y\right) = \dfrac{k}{y} f_X\left(c\log_b y\right)$ (same as what we found earlier).

Signal Power and Energy

- Consider a signal $g(t)$.
- **Energy**: $E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df$.
- **Power**: $P_g = \langle |g(t)|^2 \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |g(t)|^2 dt$.
- $g(t)$ is an **energy signal** iff $0 < E_g < \infty$.
  - In which case, $P_g = 0$.
- $g(t)$ is a **power signal** iff $0 < P_g < \infty$.
  - In which case, $E_g = \infty$.
- If $g(t)$ is periodic with period T_0, then
  - $g(t)$ is a power signal,
  - $P_g = \frac{1}{T_0} \int_{T_0} |g(t)|^2 dt$,
  - $g(t)$ can be expanded in terms of complex exponential signals as $\sum_{k=-\infty}^{\infty} c_k e^{jk\frac{2\pi}{T_0}t}$ (Fourier series) and $P_g = \sum_{k=-\infty}^{\infty} |c_k|^2$ (Parseval's identity)

Power Calculation: Examples

$$g(t) = \cos(2\pi f_c t) = \frac{1}{2}e^{j2\pi f_c t} + \frac{1}{2}e^{-j2\pi f_c t}$$

$$P_g = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

$$g(t) = a(t)\cos(2\pi f_c t + \phi)$$

$$P_g = \frac{1}{2}P_a$$

Assume $A(f\text{-}f_c)$ and $A(f+f_c)$ do not overlap.

$$g(t) = \sum_k a_k(t)\cos(2\pi f_c t + \phi_k)$$

$$P_g = \frac{1}{2}\sum_k P_{a_k}$$

Assume the $A_k(f \pm f_k)$'s do not overlap.

$$g(t) = a_1\cos(2\pi f_c t + \phi_1) + a_2\cos(2\pi f_c t + \phi_2)$$

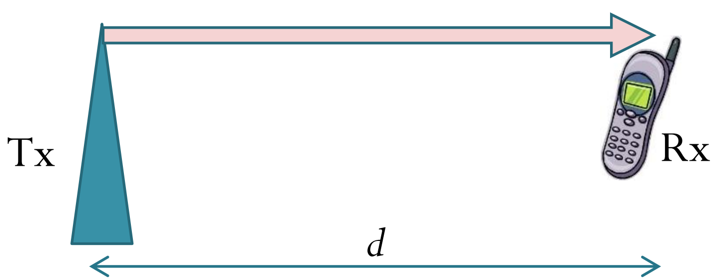
$$P_g = \frac{1}{2}a_1^2 + \frac{1}{2}a_2^2 + a_1 a_2\cos(\phi_2 - \phi_1)$$

Ray tracing

- Approximate the solution of Maxwell's equations
  - Approximate the propagation of electromagnetic waves by representing the wavefronts as simple **particles**.
  - Thus, the reflection, diffraction, and scattering effects on the wavefront are approximated using **simple geometric equations** instead of Maxwell's more complex wave equations.
- Assumption: the received waveform can be approximated by the sum of the free space wave from the transmitter plus the reflected free space waves from each of the reflecting obstacles.

$$x(t) = \sqrt{2P_t}\cos(2\pi f_c t)$$

$$y(t) = \frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{d}{c}\right)\right)$$

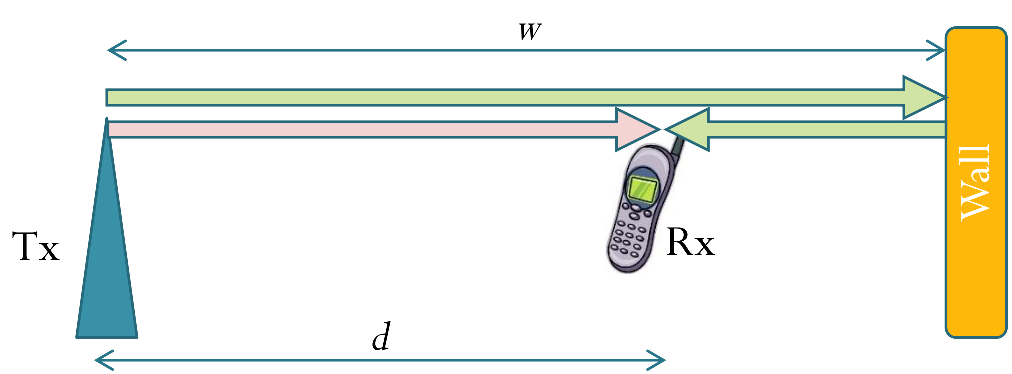


From Friis equation,

$$\alpha = \frac{\sqrt{G_{Tx}G_{Rx}}\lambda}{4\pi}.$$

Ex. One reflecting wall (1/4)

- There is a fixed antenna transmitting the sinusoid $x(t)$, a fixed receive antenna, and a single perfectly reflecting large fixed wall.
- Assume that the wall is very large, the reflected wave at a given point is the same (except for a sign change) as the free space wave that would exist on the opposite side of the wall if the wall were not present



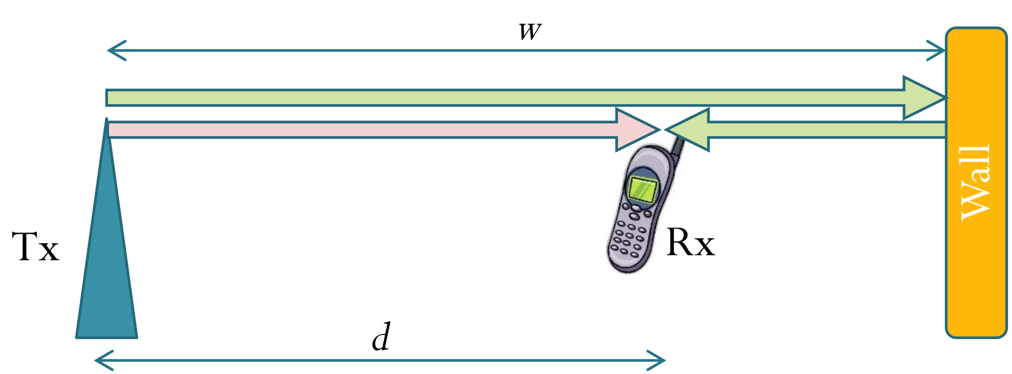
Ex. One reflecting wall (2/4)

$$x(t)=\sqrt{2P_t}\cos(2\pi f_c t)$$

$$y(t)=\frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{d}{c}\right)\right)-\frac{\alpha}{w+w-d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{w+w-d}{c}\right)\right)$$

$$=\frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{d}{c}\right)\right)-\frac{\alpha}{2w-d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{2w-d}{c}\right)\right)$$

$$=\frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{d}{c}\right)\right)+\frac{\alpha}{2w-d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{2w-d}{c}\right)-\pi\right)$$



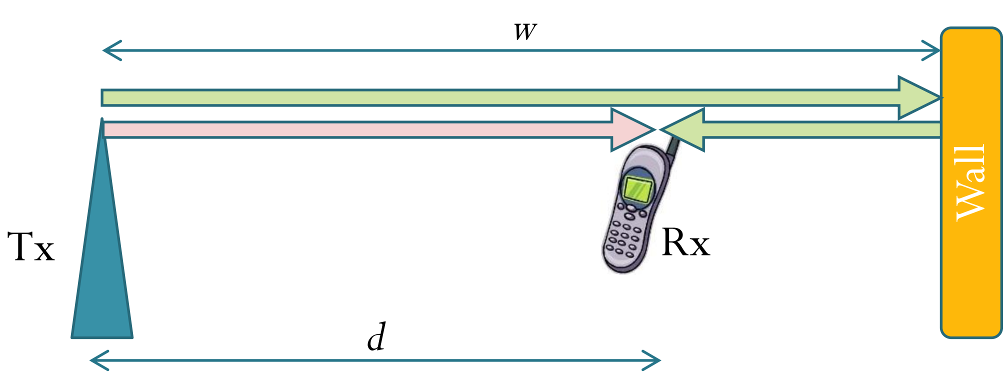
Ex. One reflecting wall (3/4)

$$y(t)=\frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{d}{c}\right)\right)+\frac{\alpha}{2w-d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{2w-d}{c}\right)-\pi\right)$$

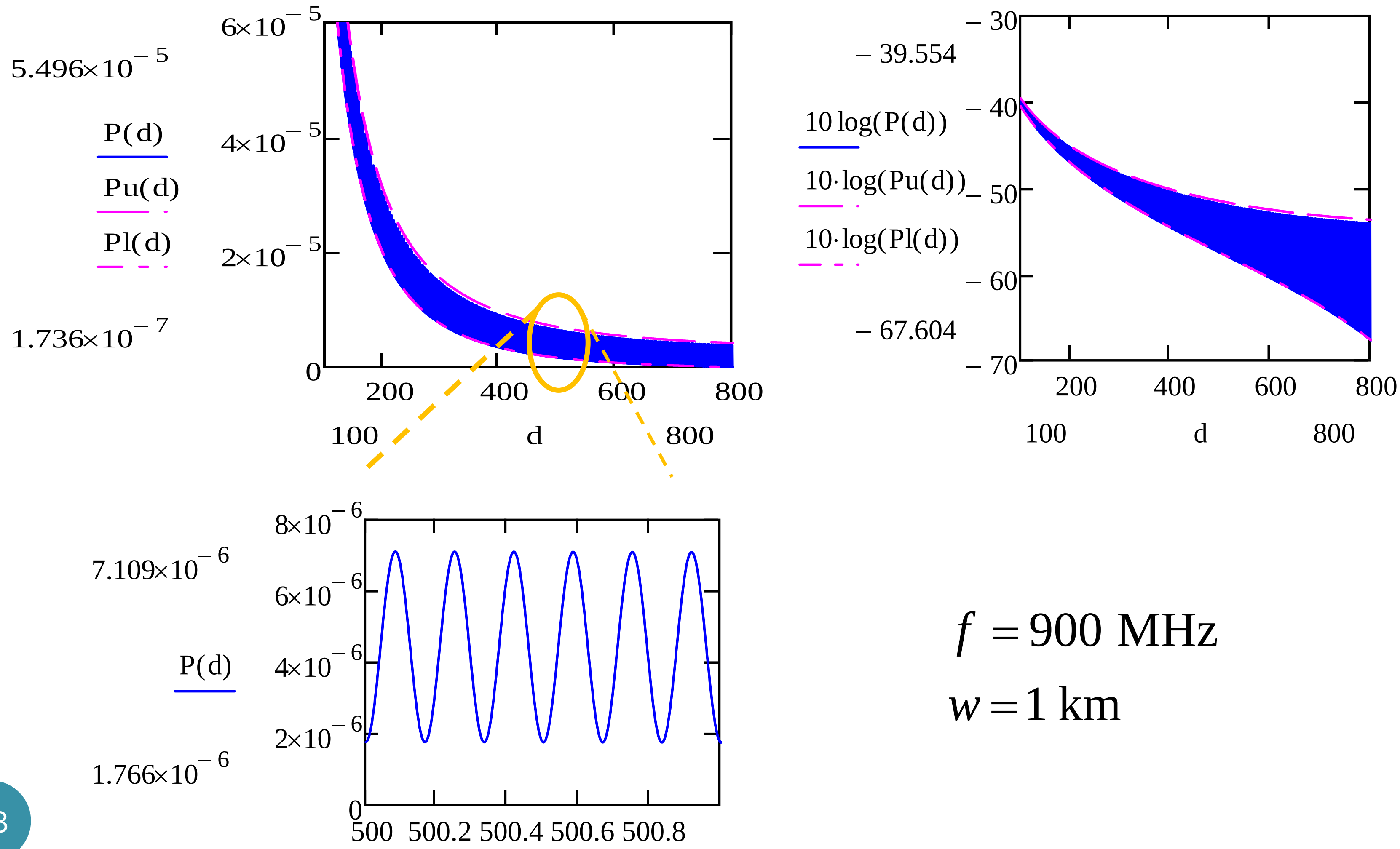
$$P_y=P_t\left(\left(\frac{\alpha}{d}\right)^2+\left(\frac{\alpha}{2w-d}\right)^2+2\frac{\alpha^2}{d(2w-d)}\cos(\Delta\phi)\right)$$

$$\Delta\phi=2\pi f_c\frac{2w-2d}{c}+\pi=2\pi\frac{1}{\lambda/2}(w-d)+\pi$$

form constructive and destructive interference pattern



Ex. One reflecting wall (4/4)



$$f = 900 \text{ MHz}$$

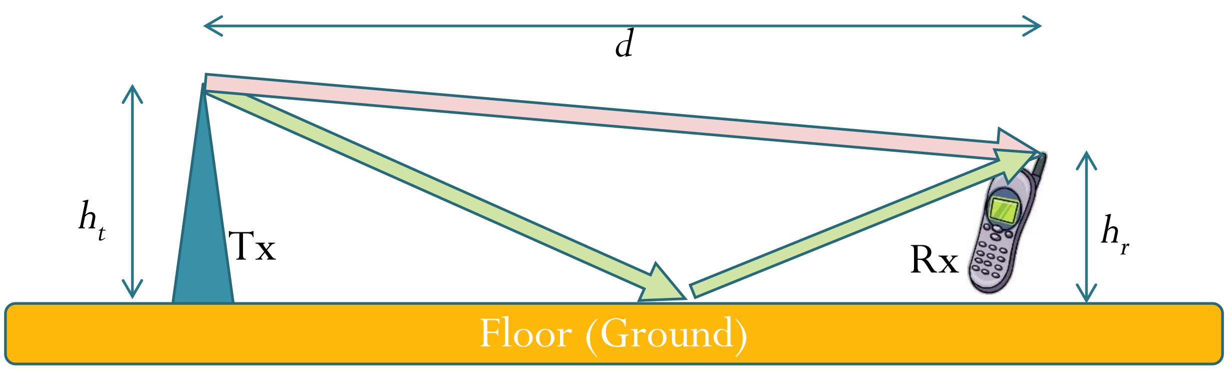
$$w = 1 \text{ km}$$

Ex. Two-Ray Model

$$\text{Delay spread} = \frac{r_2}{c} - \frac{r_1}{c}$$

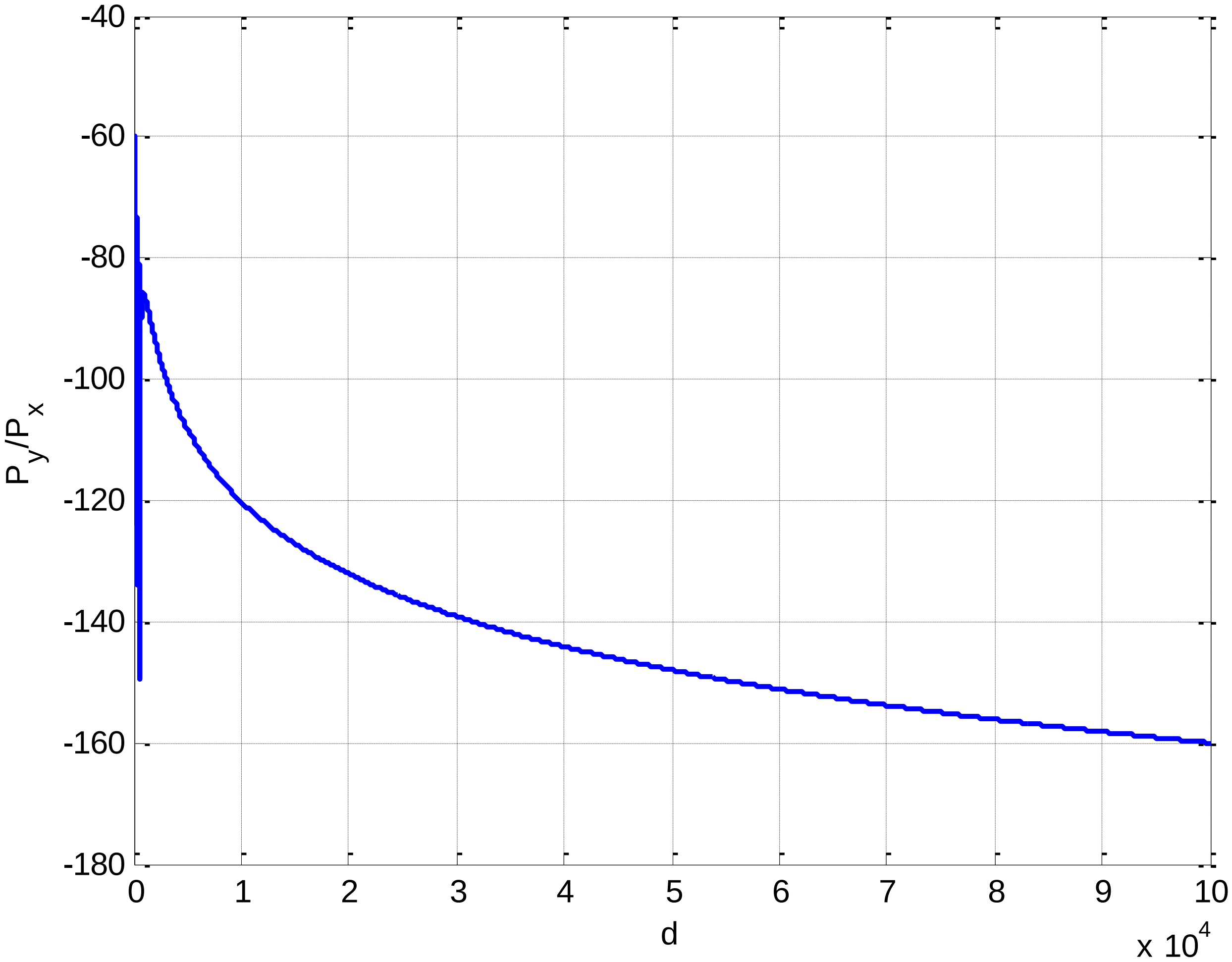
$$y(t) = \frac{\alpha}{r_1}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r_1}{c}\right)\right) - \frac{\alpha}{r_2}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r_2}{c}\right)\right)$$

$$\frac{P_y}{P_x} = \left|\frac{\alpha}{r_1}e^{-j2\pi f_c\frac{r_1}{c}} - \frac{\alpha}{r_2}e^{-j2\pi f_c\frac{r_2}{c}}\right|^2 = \left|\frac{\alpha}{r_1} - \frac{\alpha}{r_2}e^{-j2\pi f_c\frac{r_2 - r_1}{c}}\right|^2$$



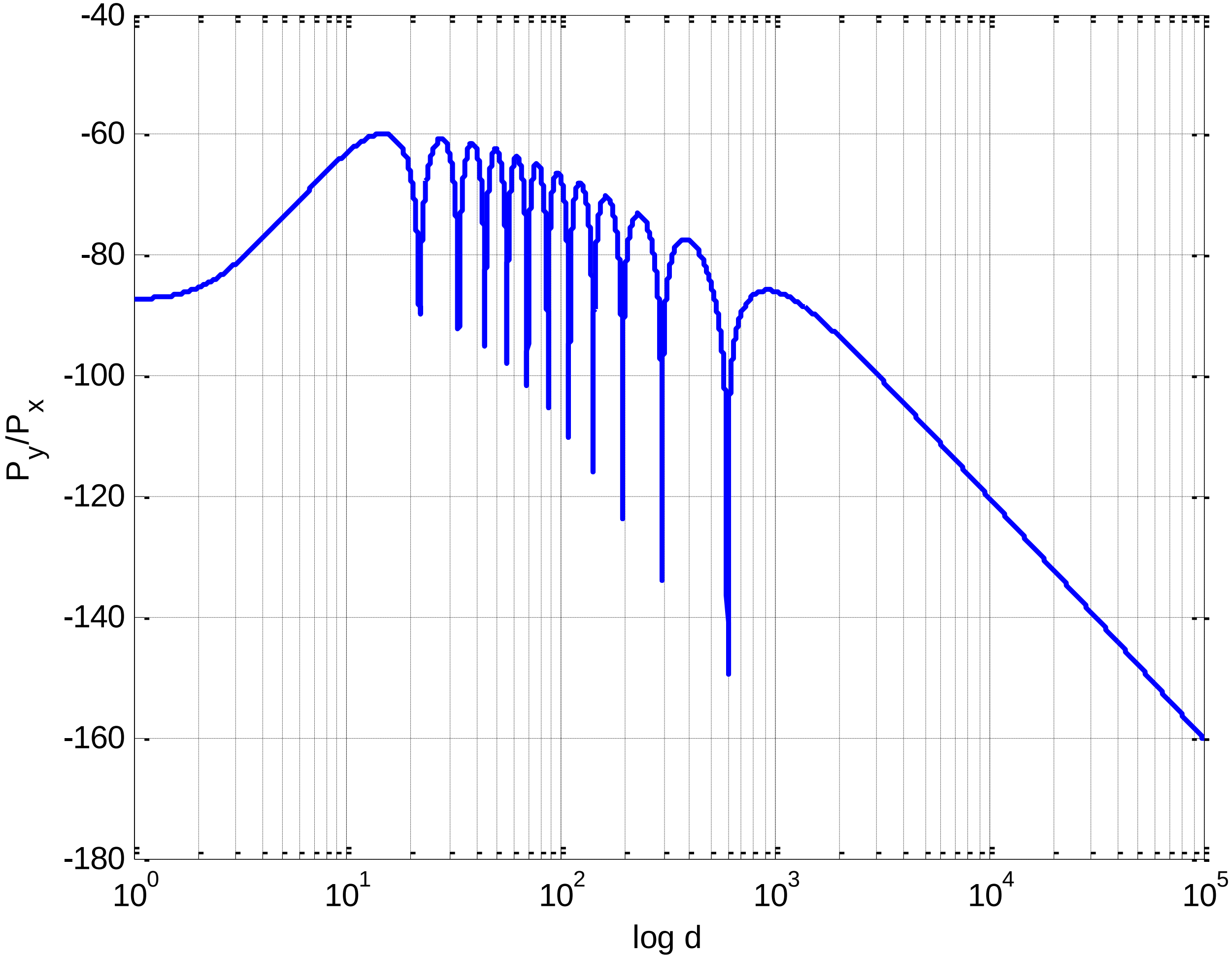
Assume ground reflection coefficient = -1.

Ex. Two-Ray Model



$$f = 900 \text{ MHz}$$
$$h_t = 50 \text{ m}$$
$$h_r = 2 \text{ m}$$

Ex. Two-Ray Model



$$f = 900 \text{ MHz}$$
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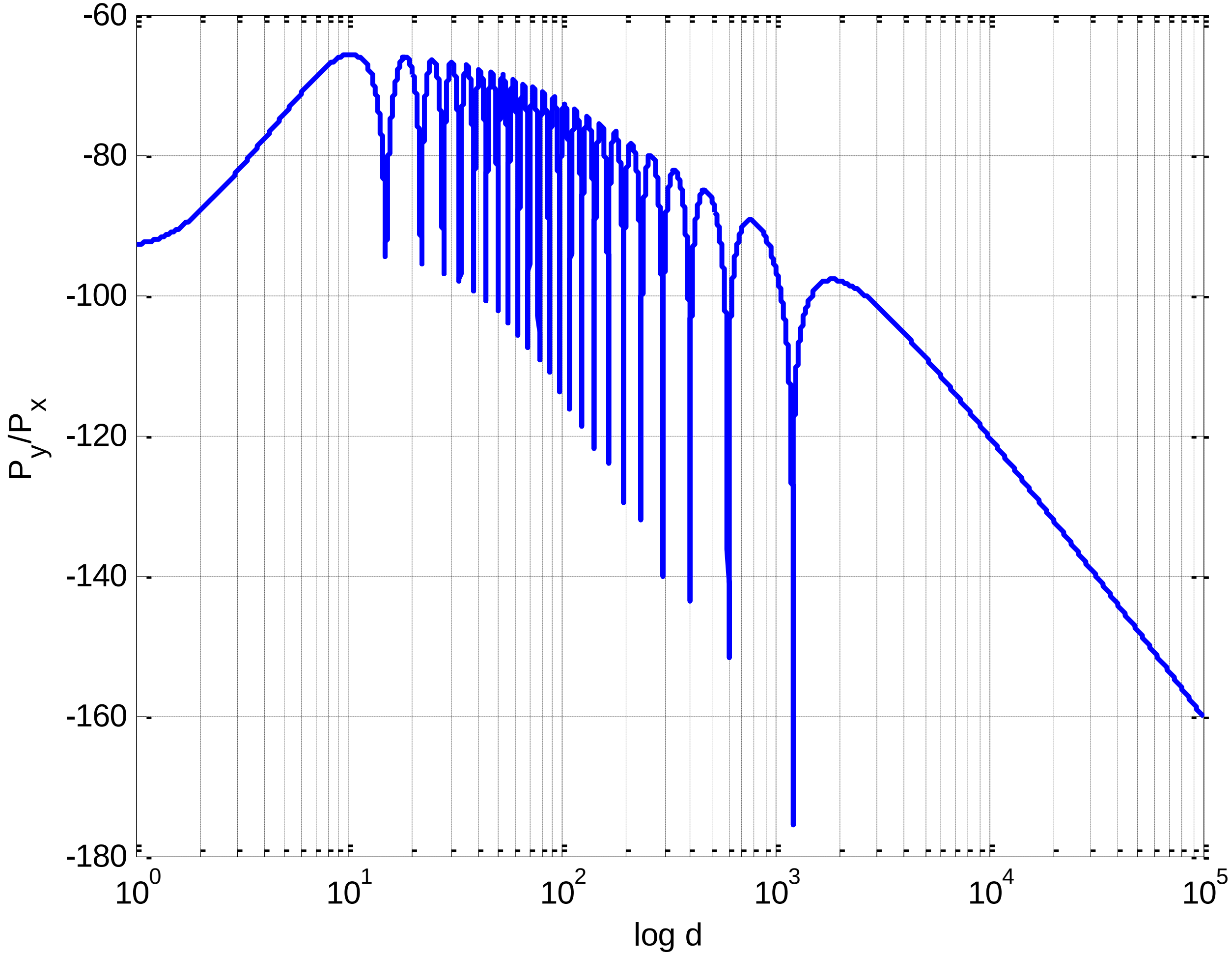
Ex. Two-Ray Model



$f = 1800$ MHz

$h_t = 50$ m

$h_r = 2$ m

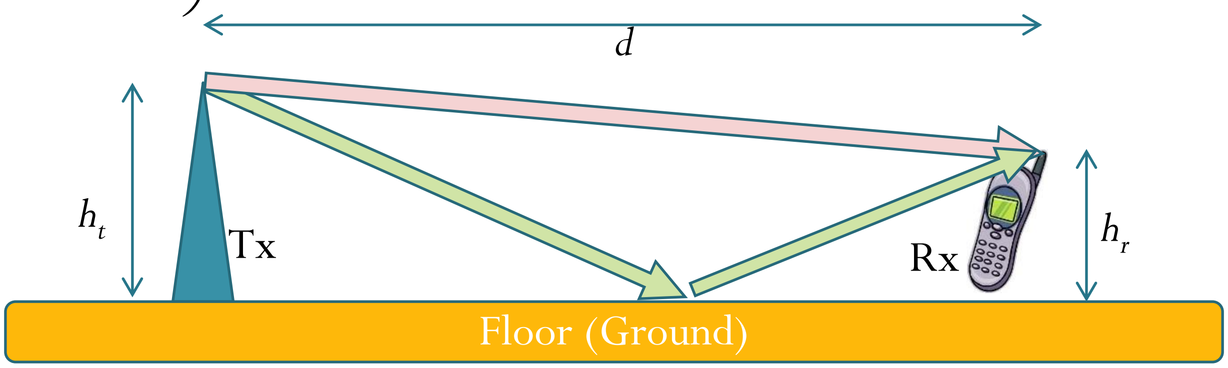
Ex. Two-Ray Model (Approximation)

$$\frac{P_y}{P_x} \approx \left| \frac{\alpha}{r_1} - \frac{\alpha}{r_2} e^{-j2\pi\frac{2h_t h_r}{\lambda}} \right|^2 \approx \frac{\alpha}{d} \left| 1 - e^{-j2\pi\frac{2h_t h_r}{\lambda d}} \right|^2$$

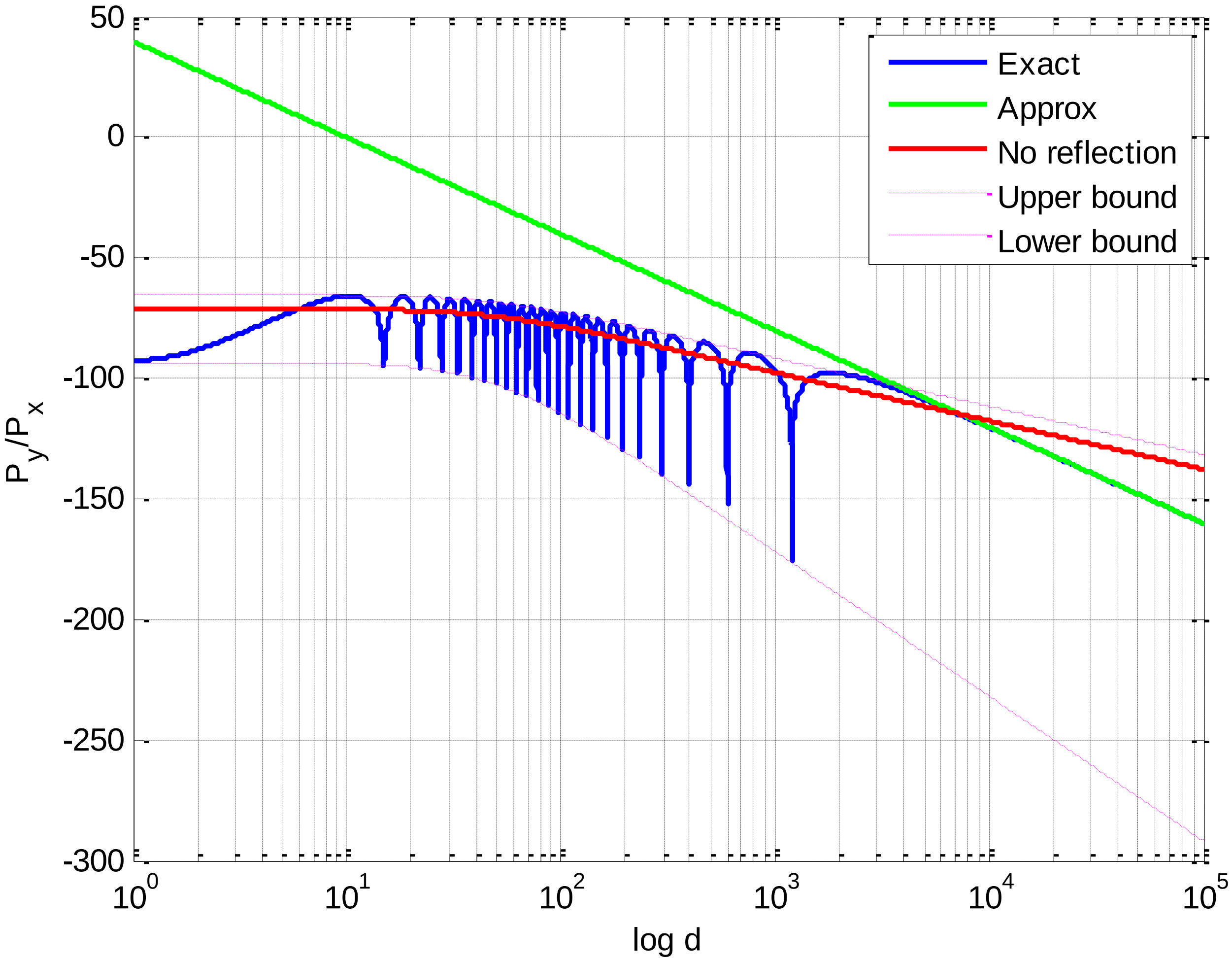
$$d \gg h_t, h_r$$

$$\approx \left(\frac{\alpha}{d}\right)^2 \left| 1 - \left(1 - j2\pi\frac{2h_t h_r}{\lambda d}\right) \right|^2 = \frac{\alpha^2}{d^2} \left| j2\pi\frac{2h_t h_r}{\lambda d} \right|^2 = \left(\frac{4\pi\alpha h_t h_r}{\lambda d^2}\right)^2$$

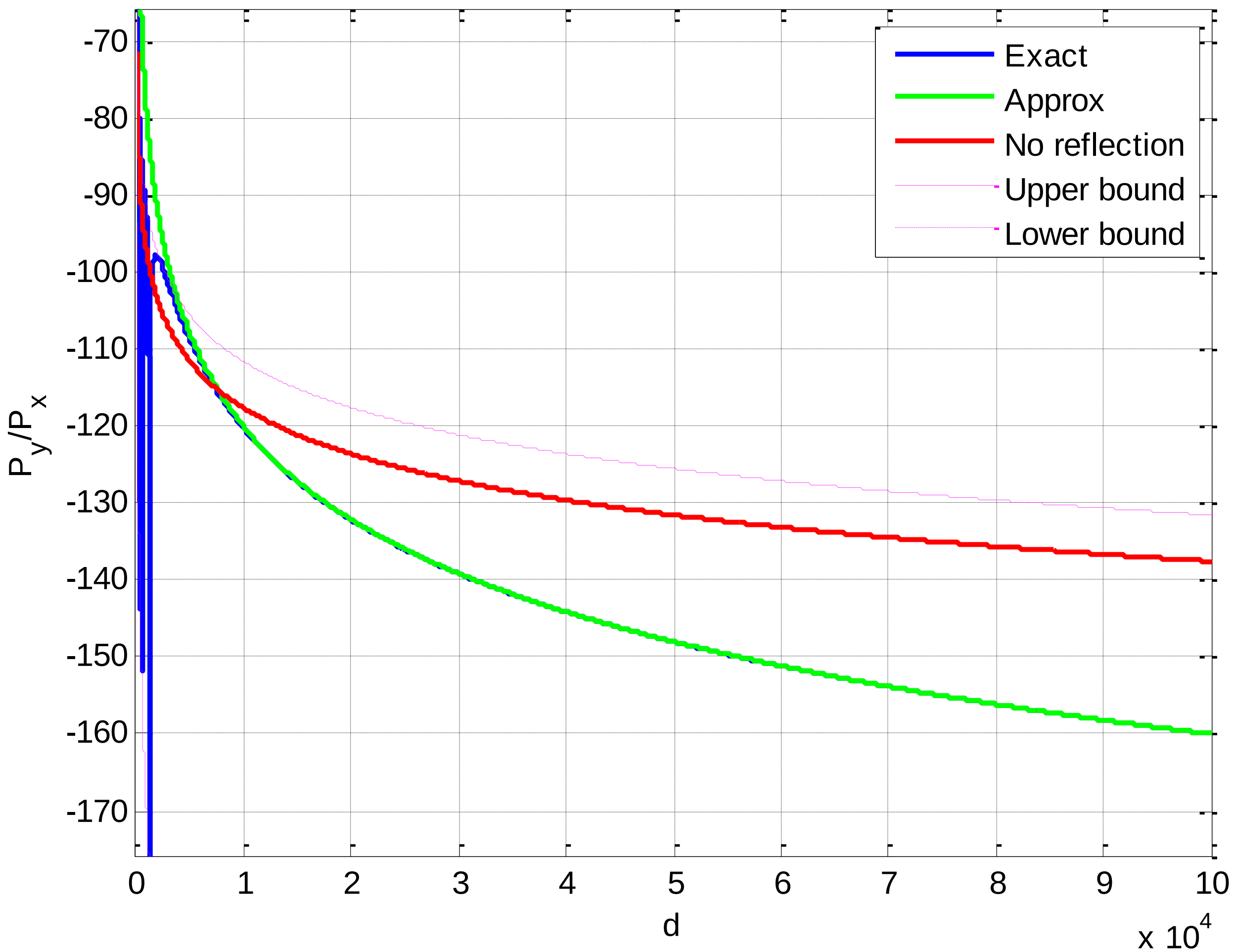
$$= \left(\frac{\sqrt{G_{Tx} G_{Rx}}\, h_t h_r}{d^2}\right)^2$$



Ex. Two-Ray Model



Ex. Two-Ray Model



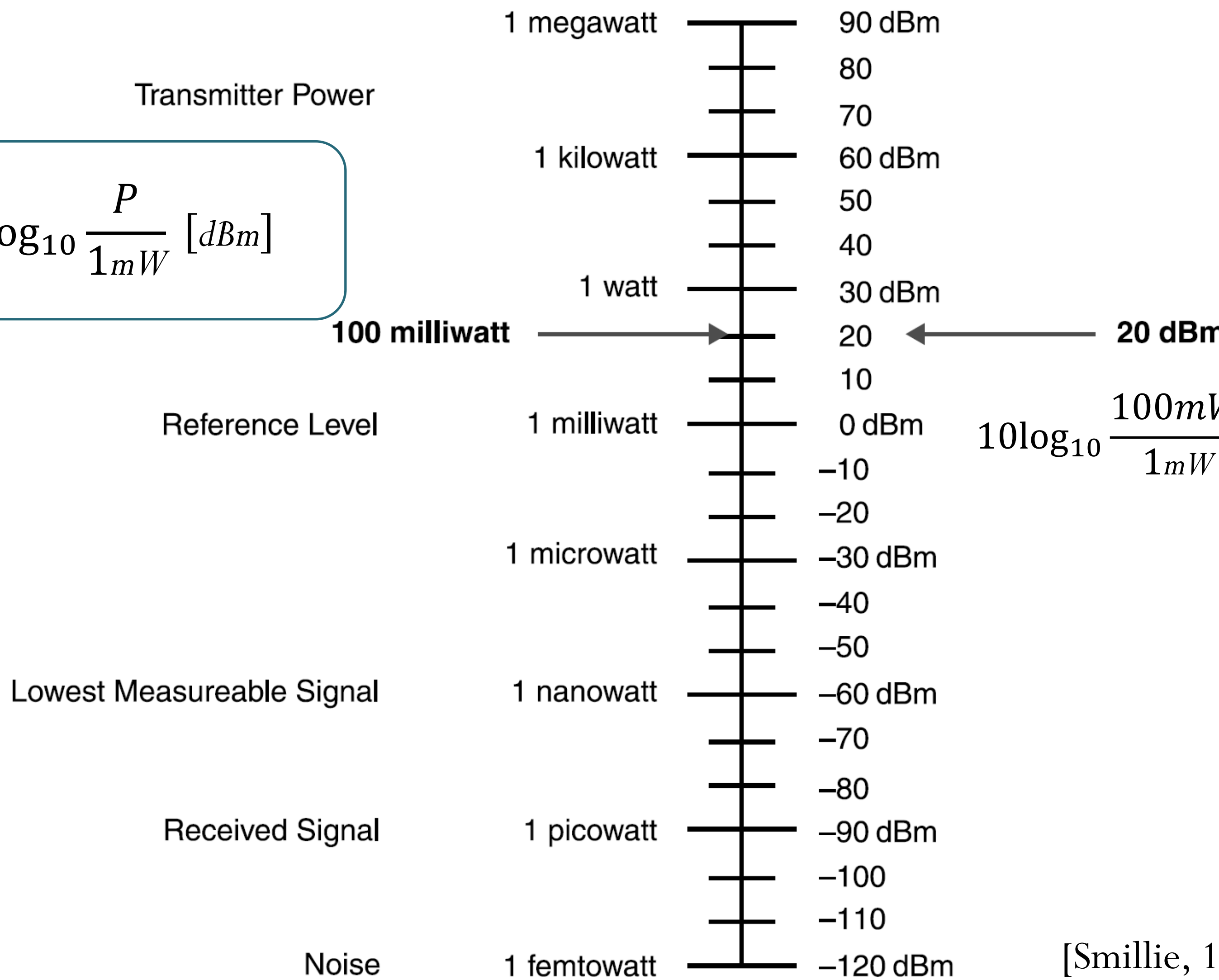
dBm

- The range of RF power that must be measured in cellular phones and wireless data transmission equipment varies from
  - hundreds of watts in base station transmitters to
  - picowatts in receivers.
- For calculations to be made, all powers must be expressed in the same power units, which is usually **milliwatts**.
  - A transmitter power of 100 W is therefore expressed as 100,000mW. A received power level of 1 pW is therefore expressed as 0.000000001mW.
- Making power calculations using decimal arithmetic is therefore complicated.
- To solve this problem, the dBm system is used.

[Scott and Frobenius, 2008, Fig 1.1]

Range of RF Power in Watts and dBm

$$P\,[W] = 10\log_{10}\frac{P}{1mW}\,[dBm]$$



$$10\log_{10}\frac{100mW}{1mW} = 20\,[dBm]$$

[Smillie, 1999, p 11]

Doppler Shift: 1D Move

- At the transmitter, suppose we have

$$\sqrt{2P_t}\cos\left(2\pi f_c t+\phi\right)$$

- At distance *r* (far enough), we have

Time to travel a distance of *r*

$$\frac{\alpha}{r}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{r}{c}\right)+\phi\right)$$

- If moving, *r* becomes *r*(*t*).
- If moving ***away*** at a constant velocity *v*, then $r(t)=r_0+vt.$

$$\frac{\alpha}{r(t)}\cos\left(2\pi f_c\left(t-\frac{r_0+vt}{c}\right)+\phi\right)=\frac{\alpha}{r(t)}\cos\left(2\pi\left(f_c-f_c\frac{v}{c}\right)t-2\pi f_c\frac{r_0}{c}+\phi\right)$$

Frequency shift

$$=\frac{v}{\lambda}$$

Review: Instantaneous Frequency

For a generalized sinusoid signal

$$A\cos\big(\theta(t)\big),$$

the **instantaneous frequency** at time t is given by

$$f(t)=\frac{1}{2\pi}\frac{d}{dt}\theta(t).$$

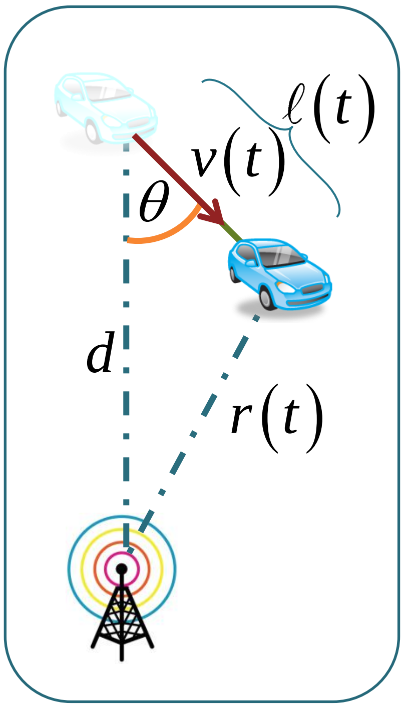
When $\theta(t)=2\pi f_c\left(t-\frac{r(t)}{c}\right)+\phi,$

$$f(t)=\frac{1}{2\pi}\frac{d}{dt}\theta(t)=f_c-\frac{f_c}{c}\frac{d}{dt}r(t)=f_c-\lambda\frac{d}{dt}r(t)$$

Frequency shift

Doppler Shift: With angle

Rx speed $= v(t)$. At time t, cover distance $\int_0^t v(\tau)d\tau$



$$r(t) = \sqrt{d^2 + \ell^2(t) - 2d\ell(t)\cos\theta}$$

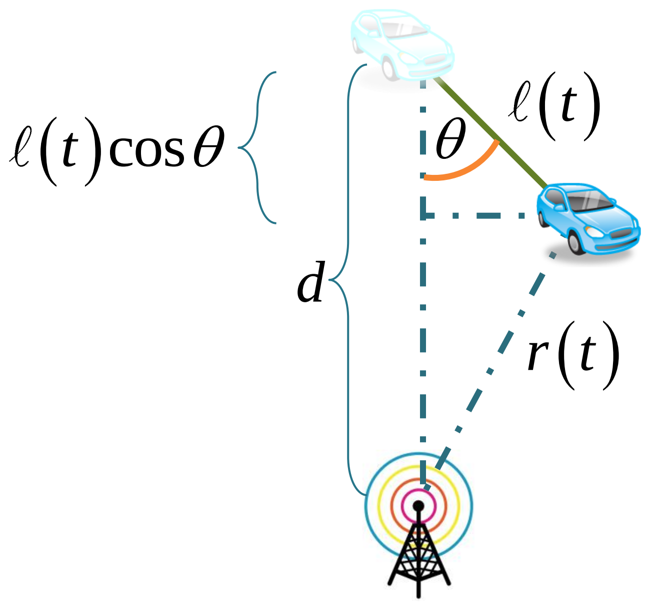
$$\frac{d}{dt}r(t) = \frac{2\ell(t) - 2d\cos\theta}{2\sqrt{d^2 + \ell^2(t) - 2d\ell(t)\cos\theta}}v(t)$$

$$\left.\frac{d}{dt}r(t)\right|_{t=0} = -\cos\theta v(0)$$

$$f_{\text{new}}(t) = f_c - \frac{1}{\lambda}\frac{d}{dt}r(t)$$

$$f_{\text{new}}(0) = f_c + \underbrace{\frac{1}{\lambda}\cos\theta v(0)}_{\textbf{Frequency shift}}$$

Doppler Shift: Approximation



$$r(t) \approx d - \ell(t)\cos\theta$$

$$\frac{d}{dt} r(t) \approx -v(t)\cos\theta$$

$$f_{\text{new}}(t) \approx f + \frac{v(t)\cos\theta}{\lambda}$$

$$\Delta f = \frac{v\cos\theta}{\lambda}$$

For typical vehicle speeds (75 Km/hr) and frequencies (around 1 GHz), it is on the order of 100 Hz

[Goldsmith, 2005, Fig 2.2]

Big Picture

Transmission impairments in cellular systems

Physics of radio propagation	Attenuation (Path Loss) Shadowing Doppler shift Inter-symbol interference (ISI) Flat fading Frequency-selective fading
Extraneous signals	Co-channel interference Adjacent channel interference Impulse noise White noise
Transmitting and receiving equipment	White noise Nonlinear distortion Frequency and phase offset Timing errors

[Myung and Goodman, 2008, Table 2.1]